THE RELATIONS ON SURFACES OF STRESS DISCONTINUITY IN THREE-DIMENSIONAL PERFECTLY RIGID-PLASTIC BODIES

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Discontinuous solutions of the equations of the theory of perfect plasticity have often been applied in the solution of many problems of plane strain, plane stress, and torsion of prismatic bars. Examples of the use of discontinuous solutions in these cases are quite well known [1 to 3].

In [4] the conditions on a surface of stress discontinuity are investigated for a three-dimensional body for states of stress corresponding to an edge of the Tresca prism. In [5] it is shown that for a convex yield surface the displacements are continuous and the plastic strain rates are zero at a surface of stress discontinuity. It should be noted that the well-known conditions at surfaces where the stresses are discontinuous were obtained for statically determinate problems.

In this paper relations are derived on surfaces of stress discontinuity for an arbitrary yield condition and the consequences of these relations are obtained for the Mises and Tresca yield conditions. The equilibrium of a regular four-sided pyramid is examined as an example.

1. In a perfectly plastic body let there exist some surface G on which the velocities u_i, the stresses σ_{ij} and the strain rates ϵ_{ij} in general suffer some discontinuity. In what follows we shall consider a surface of discontinuity G in isotropic, rigid-plastic bodies. Moreover, we shall restrict ourselves to the case in which the material on both sides of G is in the plastic state. Then the stresses σ_{ij}^+ and σ_{ij}^- on the two sides of the surface of discontinuity must satisfy the yield condition

$$f(\sigma_{ij}^+) = k, \qquad f(\sigma_{ij}^-) = k \tag{1.1}$$

From the conditions of equilibrium it follows that the traction vector on this surface must be continuous, i.e.,

$$[\sigma_{ij}]\, \nu_j = 0, \qquad [\sigma_{ij}] = \sigma_{ij}^- - \sigma_{ij}^+ \tag{1.2}$$

where ν_j is the unit vector normal to the surface of discontinuity. The strain rates in a rigid-plastic body are related to the stresses by the associated flow rule

$$\varepsilon_{ij} = {}^1\!/_2\,(u_{i,j} + u_{j,i}) = \lambda\,(\partial f / \partial \sigma_{ij}) = \lambda f_{ij} \tag{1.3}$$

where λ is an undetermined factor greater than zero, and the comma denotes partial differentiation with respect to the coordinate indicated. It follows from (1.3) that on the surface of discontinuity the following relation holds

$$[\varepsilon_{ij}] = {}^1\!/_2\,([u_{i,j}] + [u_{i,j}]) = [\lambda f_{ij}] \tag{1.4}$$

It is known [7] that surfaces of discontinuity of velocities in an incompressible material coincide with surfaces of maximum shear and can occur in an arbitrary state of stress lying on the yield surface only for the Tresca yield condition. For other yield conditions surfaces of discontinuity of velocity are possible only for quite definite combinations of the elements of the stress deviator. The stress deviator must be continuous across a surface of velocity discontinuity. It follows from the continuity of the stress deviator and from Eqs. (1.1) and (1.2) that the stresses are continuous at a surface of velocity discontinuity. Surfaces of discontinuity corresponding to a face of the Tresca yield surface constitute an exception. In this exceptional case, the direction cosines of the principal axes and also the maximum and minimum principal stresses are continuous across this surface, and only the intermediate principal stress can experience a discontinuity.

In what follows we shall consider surfaces of stress discontinuity on which the velocities are continuous. From the continuity of velocities and the geometric conditions of compatibility it follows that the jumps in strain rate at a surface of stress discontinuity G can be represented in the form

$$[\varepsilon_{ij}] = {}^1/_2\,(a_i\nu_j + a_j\nu_i), \qquad a_i = [u_{i,\,j}]\,\nu_j \tag{1.5}$$

Choosing the canonical coordinate system $\nu_1 = \nu_2 = 0,\ \nu_3 = 1$, we find from Eqs. (1.2) and (1.5) and from the condition of incompressibility that

$$[\varepsilon_{11}] = [\varepsilon_{22}] = [\varepsilon_{33}] = [\varepsilon_{12}] = 0, \qquad [\sigma_{13}] = [\sigma_{23}] = [\sigma_{33}] = 0 \tag{1.6}$$

It follows from (1.6) that

$$[\sigma_{ij}]\,[\varepsilon_{ij}] = 0$$

On the other hand,

$$[\sigma_{ij}]\,[\varepsilon_{ij}] = (\sigma_{ij}^- - \sigma_{ij}^+)\,\varepsilon_{ij}^- + (\sigma_{ij}^+ - \sigma_{ij}^-)\,\varepsilon_{ij}^+ \geqslant 0 \tag{1.7}$$

For convex yield surfaces, the right-hand side of Eq. (1.7) goes to zero only for $\sigma_{ij}^- = \sigma_{ij}^+$ or for $\varepsilon_{ij}^- = \varepsilon_{ij}^+ = 0$. Therefore, for convex yield surfaces the strain rates go to zero on a surface of stress discontinuity.

The relations (1.1) and (1.2) do not determine all the limitations which must be imposed on the state of stress at the surface G. The associated flow rule (1.3) must be used to determine the remaining relations. Let us first examine the special case when σ_{ij}^- and σ_{ij}^+ lie on a plane part of the yield surface. In this case the right-hand side of Eq. (1.7) is identically zero and the strain rates can be discontinuous.

We have from Eqs. (1.4) and (1.5) that

$${}^1/_2\,(a_i\nu_j + a_j\nu_i) = [\lambda f_{ij}] \tag{1.8}$$

Equating the indices i and j in (1.8), we obtain for incompressible bodies

$$a_i\nu_i = [\lambda f_{ii}] = 0 \tag{1.9}$$

Multiplying Eq. (1.8) through by ν_j and taking (1.9) into account, we have

$$a_i = 2\,[\lambda f_{ij}]\,\nu_j \tag{1.10}$$

Eliminating the quantities a_i from Eqs. (1.8) with the aid of (1.10) we have

$$[\lambda f_{il}]\,\nu_l\nu_j + [\lambda f_{jl}]\,\nu_l\nu_i = [\lambda f_{ij}] \tag{1.11}$$

Only three of the six relations (1.11) are independent, since after contraction with the Kronecker tensor δ_{ij} and with ν_i these equations reduce to a single form. The three independent relations of (1.11) together with (1.1) and (1.2) form a complete system of equations for the determination λ^-/λ^+ and σ_{ij}^- if σ_{ij}^+ and the position of the surface are known. Here λ^-/λ^+ must be positive.

Let us show that Eqs. (1.11) also hold at a surface of stress discontinuity on which the strain rates ϵ_{ij}^- and ϵ_{ij}^+ go to zero. In this case, however, λ must be taken as some unknown quantity differing from the factor in the associated flow rule. The meaning of this quantity will be obvious in the course of the exposition.

We remark that if $\epsilon_{ij}^- = \epsilon_{ij}^+ = 0$ on the surface G, then it follows from the associated flow rule (1.3) that $\lambda^+ = \lambda^- = 0$, in which case the relations (1.8) and (1.11) become identities, and from (1.10), $a_i = 0$. Therefore, the velocities and their first derivatives are continuous across a surface of stress discontinuity.

To determine the limitations which the associated flow rule imposes on the quantities σ_{ij}^+ and σ_{ij}^-, the relations (1.8) in this case must be differentiated with respect to some direction x_l which is not tangent to the surface of discontinuity G. Here, since $\lambda^+ = \lambda^- = \sigma$ on the surface, G, we have

$$[\varepsilon_{ij,\, l}] = [\lambda_{,\, l} f_{ij}] \tag{1.12}$$

The geometric conditions of compatibility of second order for the quantities u_i, which are continuous along with their first derivatives, have the form

$$[u_{i,\, jl}] = b_i \nu_j \nu_l, \qquad b_i = [u_{i,\, mn}]\, \nu_m \nu_n \tag{1.13}$$

Using Eqs. (1.13), we have from (1.12) that

$$b_i \nu_l \nu_j + b_j \nu_l \nu_i = 2\, [\lambda_{,l}\ f_{ij}] \tag{1.14}$$

By comparing Eqs. (1.8) and (1.14), it is easy to see that they agree if the quantities $b_i \nu_l$ and $\lambda_{,l}$ are replaced by a_i and λ, respectively. It follows from this that in the case under consideration relations analogous to (1.11) hold at the surface of discontinuity and that all the conclusions which follow from them are valid.

If the first derivatives of the strain rates are also zero in a surface of stress discontinuity, then $\lambda_{,l} = 0$ and the relations (1.14) become identities. In this case, the associated flow rule should be differentiated twice, and by repeating all the arguments we again obtain that equations analogous to (1.11) hold on a surface of stress discontinuity. If the second derivatives of ϵ_{ij} vanish, then the differentiation should be carried out three times, etc.

It is not possible for all the derivatives of ϵ_{ij} to vanish, since then $\epsilon_{ij} = 0$ in a region of plastic flow. Therefore, Eqs. (1.11) indeed constitute additional limitations on the quantities σ_{ij}^- and σ_{ij}^+ for any nonconcave yield surface. We remark that the quantities λ^- and λ^+ are positive in the vicinity of the surface of discontinuity. Therefore, the leading term in the Taylor expansions of λ^+ and λ^- in x_l must be positive. It follows that the ratio λ^-/λ^+ must have the sign of $(-1)^n$, where n is the number of differentiations needed to obtain Eqs. (1.11).

In the canonical system of coordinates $\nu_1 = \nu_2 = 0,\ \nu_3 = 1$, the relations on the surface of discontinuity (1.1), (1.2), and (1.11) simplify and have the form

$$[\sigma_{i3}] = 0, \quad [f(\sigma_{ij})] = 0, \quad [\lambda f_{11}] = [\lambda f_{22}] = [\lambda f_{12}] = 0 \tag{1.15}$$

2. Let us examine the consequence of the relations (1.15) for the Mises yield condition

$$f(\sigma_{ij}) = s_{ij} s_{ij} = k^2$$

In this case the relations (1.15) assume the form

$$[\sigma_{i3}] = 0, \qquad [s_{ij} s_{ij}] = 0, \qquad [\lambda s_{11}] = [\lambda s_{22}] = [\lambda s_{12}] = 0 \tag{2.1}$$

It follows from (2.1) that

$$\{1 - (\lambda^- / \lambda^+)^2\}\,(s_{11}^2 + s_{22}^2 + s_{33}^2 + 2 s_{12}^2) = 0 \tag{2.2}$$

Eq. (2.2) will be satisfied if one of the following relations holds:

$$s_{11} = s_{22} = s_{33} = s_{13} = 0, \qquad \lambda^+ = \lambda^-,\ \lambda^+ = -\lambda^-$$

In the first two cases continuity of the stresses follows from (2.1). Thus, at a surface of stress discontinuity $\lambda^+ = -\lambda^-$. Here we have from (2.1)

$$\sigma_{11}^- = 2\sigma_{33}^+ - \sigma_{11}^+, \qquad \sigma_{22}^- = 2\sigma_{33}^+ - \sigma_{22}^+, \qquad \sigma_{33}^- = \sigma_{33}^+$$
$$\tau_{12}^- = -\tau_{12}^+, \qquad \tau_{13}^- = \tau_{13}^+, \qquad \tau_{23}^- = \tau_{23}^+ \tag{2.3}$$

If l_i, m_i, n_i are the direction cosines of the principal axes of the stress tensor, then

$$\sigma_{ij} = \sigma_1 l_i l_j + \sigma_2 m_i m_j + \sigma_3 n_i n_j \tag{2.4}$$

Substituting (2.4) into (2.3) and taking account of the fact that

$$l_i l_j + m_i m_j + n_i n_j = \delta_{ij} \tag{2.5}$$

we obtain a system of twelve equations for the determination of σ_1^-, σ_2^-, σ_3^-, l_i^-, m_i^-, n_i^-. The solution of this system has the form

$$\begin{aligned} &\sigma_1^- = 2\sigma_{33} - \sigma_1^+, && \sigma_2^- = 2\sigma_{33} - \sigma_2^+, && \sigma_3^- = 2\sigma_{33} - \sigma_3^+ \\ &l_1^- = \pm l_1^+, && m_1^- = \pm m_1^+, && n_1^- = \mp n_1^+ \\ &l_2^- = \pm l_2^+, && m_2^- = \pm m_2^+, && n_2^- = \mp n_2^+ \\ &l_3^- = \mp l_3^+, && m_3^- = \mp m_3^+, && n_3^- = \pm n_3^+ \end{aligned} \tag{2.6}$$

It follows from Eqs. (2.6) that the pairs of principal axes on the two sides of G having the same notation, (i.e., l_i^+ and l_i^-, etc.) make equal angles with the surface of discontinuity G and are coplanar with the normal to this surface. The deviatoric components of the principal stresses have opposite signs. Therefore, the states of stress on opposite sides of the surface of discontinuity G correspond to diametrically opposite points on the yield locus in the deviatoric plane.

3. The analysis of the relations (1.15) is somewhat more complicated for the Tresca yield condition. Let the principal stresses numbered so that σ_1 is intermediate between σ_2 and σ_3. Then the yield condition has the form

$$\sigma_2 - \sigma_3 = \pm 2k \tag{3.1}$$

It follows from Eqs. (1.15), (2.4) and (2.5) that

$$[\sigma_{33}] = [\sigma_1 l_3^2 + \sigma_2 m_3^2 + \sigma_3 n_3^2] = 0, \quad [\sigma_{13}] = [\sigma_1 l_1 l_3 + \sigma_2 m_1 m_3 + \sigma_3 n_1 n_3] = 0$$
$$[\sigma_{23}] = [\sigma_1 l_2 l_3 + \sigma_2 m_2 m_3 + \sigma_3 n_2 n_3] = 0 \tag{3.2}$$

$$[\varepsilon_{11}] = [\lambda(m_1^2 - n_1^2)] = 0, \quad [\varepsilon_{22}] = [\lambda\,(m_2^2 - n_2^2)] = 0$$
$$[\varepsilon_{12}] = [\lambda\,(m_1 m_2 - n_1 n_2)] = 0 \tag{3.3}$$

The relations (3.1) to (3.3) are invariant with respect to a rotation of the coordinate system about the third axis. Let us perform such a rotation of coordinates so that σ_{13}^+ goes to zero. Then it follows from (3.2) that $\sigma_{13}^- = 0$. Taking this into account, we obtain from (3.1) and (3.2) that

$$\frac{m_3^+ n_3^+}{l_1^+} = \frac{m_3^- n_3^-}{l_1^-} \tag{3.4}$$

After elimination of λ^-/λ^+ from Eqs. (3.3), these are satisfied if, and only if, the following equalities hold:

$$\frac{m_1^- - n_1^-}{m_2^- - n_2^-} = \frac{m_1^+ - n_1^+}{m_2^+ - n_2^+}, \qquad \frac{m_1^- + n_1^-}{m_2^- + n_2^-} = \frac{m_1^+ + n_1^+}{m_2^+ + n_2^+} \tag{3.5}$$

or

$$\frac{m_1^- - n_1^-}{m_2^- - n_2^-} = \frac{m_1^+ + n_1^+}{m_2^+ + n_2^+}, \qquad \frac{m_1^- + n_1^-}{m_2^- + n_2^-} = \frac{m_1^+ - n_1^+}{m_2^+ - n_2^+} \tag{3.6}$$

Eqs. (3.4) and (3.5) will be satisfied if we set

$$l_i^- = \pm l_i^+, \qquad m_i^- = \pm m_i^+, \qquad n_i^- = \pm n_i^+ \tag{3.7}$$

and the system (3.2, 3.4, 3.6) corresponds to the system (2.6).

We shall now show that there are no other solutions of the system of equations (3.1, 3.2, 3.3). In order to do this, we carry out a rotation of coordinates so that

$$[\varepsilon_{13}] = [\lambda\,(m_1 m_3 - n_1 n_3)] = 0 \tag{3.8}$$

It follows now from the relations (3.3, 3.8) that Eqs.

$$\frac{m_1^- + n_1^-}{m_3^- + n_3^-} = \frac{m_1^+ + n_1^+}{m_3^+ + n_3^+}, \qquad \frac{m_1^- - n_1^-}{m_3^- - n_3^-} = \frac{m_1^+ - n_1^+}{m_3^+ - n_3^+} \tag{3.9}$$

or

$$\frac{m_1^- - n_1^-}{m_3^- - n_3^-} = \frac{m_1^+ + n_1^+}{m_3^+ + n_3^+}, \qquad \frac{m_1^- + n_1^-}{m_3^- + n_3^-} = \frac{m_1^+ - n_1^+}{m_3^+ - n_3^+} \tag{3.10}$$

are satisfied.

Since (2.6) and (3.7) are solutions of the system (3.1), (3.3), they must be contained in combinations of the relations (3.3), (3.9), and (3.10). For this it is necessary that the following Eq. hold for the rotation of coordinates referred to above:

$$(m_1 n_3 - m_3 n_1)(m_1 m_3 - n_1 n_3) = 0 \tag{3.11}$$

We can verify that in satisfying the relation (3.11), the system of equations (3.1) to (3.3) has only the two solutions (2.6) and (3.7). The analysis of the solution (2.6) has been carried through above. The same conclusions follow from this solution for the Tresca yield condition as for the Mises yield condition. Therefore, we shall only give an analysis of the consequences of Eqs. (3.7).

Since in accordance with the solution (3.7), the direction cosines are continuous at the surface of discontinuity, the system can be transformed into

$$\begin{aligned} [\sigma_1] l_3^2 + [\sigma_2] m_3^2 + [\sigma_3]\, n_3^2 &= 0 \\ [\sigma_1]\, l_1 l_3 + [\sigma_2]\, m_1 m_3 + [\sigma_3]\, n_1 n_3 &= 0 \\ [\sigma_1]\, l_2 l_3 + [\sigma_2]\, m_2 m_3 + [\sigma_3]\, n_2 n_3 &= 0 \end{aligned} \tag{3.12}$$

The system of equations (3.12) has a nontrivial solution if

$$l_3 m_3 n_3 = 0 \tag{3.13}$$

That is, one or two of the principal axes lie in the plane tangent to the surface of discontinuity. Analysis of the relations (3.1), (3.12), and (3.13) leads to the system of equations

$$l_3 = 0, \quad m_3 \neq 0, \quad n_3 \neq 0, \quad [\sigma_2] = [\sigma_3] = 0, \quad [\sigma_1] \neq 0 \tag{3.14}$$

$$l_3 \neq 0, \quad m_3 = 0, \quad n_3 \neq 0, \quad [\sigma_1] = [\sigma_3] = 0, \quad [\sigma_2] = \pm 4k \tag{3.15}$$

$$l_3 = m_3 = 0, \quad n_3 = 1, \quad [\sigma_3] = [\sigma_2] = 0, \quad [\sigma_1] \neq 0 \tag{3.16}$$

$$l_3 = m_3 = 0, \quad n_3 = 1, \quad [\sigma_3] = 0, \quad [\sigma_2] = \pm 4k, \quad [\sigma_1] \neq 0 \tag{3.17}$$

$$m_3 = n_3 = 0, \quad l_3 = 1, \quad [\sigma_1] = 0, \quad [\sigma_2] = [\sigma_3] \neq 0 \tag{3.18}$$

$$m_3 = n_3 = 0, \quad l_3 = 1, \quad [\sigma_1] = 0, \quad [\sigma_2] = [\sigma_3] \pm 4k \tag{3.19}$$

Here Eqs. (3.15), (3.17), and (3.19) hold if the points representing the states of stress on the two sides of the surface of stress discontinuity are on opposite faces of the yield surface. The solutions (3.16) and (3.18) correspond to a single face of the yield surface.

We remark that on the faces of the Tresca surface a discontinuity in the plastic strain rates is possible when the direction cosines of the principal stresses are continuous; Eqs. (3.1) to (3.3) will then be satisfied.

The analysis of the possible surfaces of stress discontinuity for a state of stress corresponding to an edge of the Tresca prism has been carried out in detail in [4 to 6]. In this case the relations of the theory of perfect plasticity are statistically determinate and Eqs. (1.11) do not impose any limitations on the possible jumps of stress. The results of [4] are in accord with the relations (2.6).

In conclusion we show that the relations (2.6) will hold at a surface of discontinuity of stress for the case of an incompressible 'normal' isotropic body, i.e., one in which the yield condition is not altered by a change in sign of the stress deviator. In this case the relations (1.5) can be written in the form

$$[\sigma_1 l_i l_3 + \sigma_2 m_i m_3 + \sigma_3 n_i n_3] = 0, \qquad [f(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|)] = 0$$

$$\left[\lambda\left(\frac{\partial f}{\partial \sigma_1} l_1^2 + \frac{\partial f}{\partial \sigma_2} m_1^2 + \frac{\partial f}{\partial \sigma_3} n_1^2\right)\right] = 0 \qquad (3.20)$$

$$\left[\lambda\left(\frac{\partial f}{\partial \sigma_1} l_1 l_2 + \frac{\partial f}{\partial \sigma_2} m_1 m_2 + \frac{\partial f}{\partial \sigma_3} n_1 n_2\right)\right] = 0$$

$$\left[\lambda\left(\frac{\partial f}{\partial \sigma_1} l_2^2 + \frac{\partial f}{\partial \sigma_2} m_2^2 + \frac{\partial f}{\partial \sigma_3} n_2^2\right)\right] = 0$$

We note that $\sigma_i^- = -\sigma_i^+$, then

$$(\partial f / \partial \sigma_i)^- = -(\partial f / \partial \sigma_i)^+$$

and the relations (2.6) satisfy the system (3.20).

4. As an example, let us examine the equilibrium of a regular four-sided pyramid

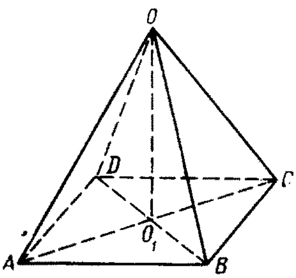

Fig. 1

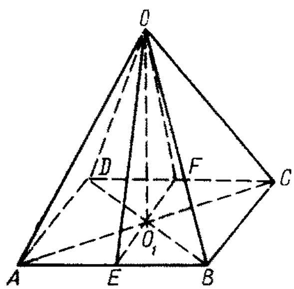

Fig. 2

(Fig. 1). We shall assume that a normal pressure p acts on the faces BCO and DOA, and that the faces AOB and DOC are free of load. We take the planes AOC and DOB as the surfaces of discontinuity of stress and examine the two adjacent regions O_1OCB and AOO_1B. We assume that there is a uniform state of stress in each of these zones. In the region AOO_1B, let the quantity $\sigma_3 = 0$, and in the region COO_1B — $\sigma_3 = p$. We assume that the state of stress in the pyramid satisfies the Tresca yield condition (3.1).

The cosines of the angles between the principal axes in the region AOO_1B and the normal to the plane OO_1B have the form

$$l = 1 / \sqrt{2}, \qquad m = \tfrac{1}{2}\sqrt{2}\sin\gamma, \qquad n = \tfrac{1}{2}\sqrt{2}\cos\gamma \qquad (4.1)$$

where 2γ is the angle between the faces AOB and DOC.

Using the relations (2.6, 3.1 and 4.1) we obtain the following expression for the limiting pressure:

$$p = \sigma_1 - 2k\sin^2\gamma$$

The maximum pressure will occur for $\sigma_1 = -2k$ and is

$$p_{max} = -2k(1 + \sin^2 \gamma)$$

which agrees with the formula obtained in [4].

For the Mises yield condition the maximum limiting pressure p is

$$p = -{}^2/_3 \sqrt{3}\, k \sqrt{(1 + \sin^2 \gamma)^2 + 3}$$

We now assume that the normal pressure acts only on the face OBC and that the faces ABO, DOC, and AOD are free of load (Fig. 2). We take the planes ACO, DOB, and EOF as the surfaces of stress discontinuity. Presuming as above that in all zones there is a uniform state of stress satisfying the Tresca yield condition, we obtain that on the surface of discontinuity EOF the direction cosines of the principal axes are continuous. From the relations (2.6), (3.1), (3.17), and (4.1), we obtain the expression $p = -4k \sin^2 \gamma$ for the limiting pressure.

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